Homework No: 1

1-11 d) $T_a = \frac{1}{5}$, $T_b = \frac{2}{17}$

$T_d = ?$ $\frac{n_1}{n_2} = \frac{T_a}{T_b} = \frac{17}{10} = \frac{\frac{1}{5}}{2/17} \Rightarrow T_d = n_1 T_b = n_2 T_a$

$\Rightarrow T_d = 17 \cdot \frac{2}{17} = 10 \times \frac{1}{5} = \boxed{2}$ ans.

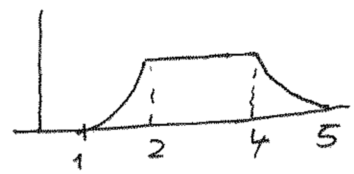
f) $T_b = \frac{2}{17}$, $T_c = \frac{2}{19}$ $\Rightarrow \frac{T_b}{T_c} = \frac{n_1}{n_2} = \frac{\frac{2}{17}}{2/19} = \frac{19}{17}$

$19\, T_c = 17\, T_b = T_f = 19 \times \frac{2}{19} = 2 = 17 \times \frac{2}{17} = \boxed{2 \text{ sec.}}$ answ

1-21 a) $x_1(t) = 2 u_{-3}(t-1)\, u(2-t)$

$+ u(t-2) - u(t-4)$

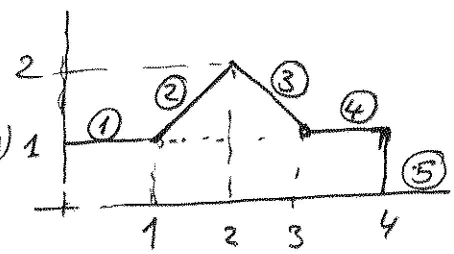
$+ 2u_{-3}(5-t)\, u(t-4))$



or $x = 2u_{-3}(t-1)\, u(2-t) + u(t-2)\, u(4-t))$

$+ 2u_{-3}(5-t)\, u(t-4)$

1-22(a)



① $= u(t)$

② $= u(t) + f(t) = r(t-1) + u(t-1)$

$f(t) = t-1 = r(t-1)$

$u(t) + r(t-1) + f(t) =$ ③

③ $= -t + 4$

$u(t) + r(t-1) + f(t) = -t+4$

$1 + t - 1 + t - 4 = -f(t) = 2t - 4 = 2r(t-2)$

$u(t) + r(t-1) - 2r(t-2) + f(t) =$ ④ $= u(t-3)$

$1 + t - 1 - 2(t-2) + f(t) = 1$

$t - 2t + 4 - 1 = -f(t) = -t + 3$

⑤ $= u(t) + r(t-1) - 2r(t-2) + r(t-3) + f(t) = 0 \Rightarrow f(t) = t-3 \Rightarrow -f(t) = \cancel{t} + \cancel{t} - \cancel{t} - 2t + 4 + \cancel{t} - 3$
$- \cancel{t} + \cancel{t} = 1 = u(t-4)$

$\therefore$ $x_1(t) = u(t) + r(t-1) - 2r(t-2) + r(t-3) - u(t-4))$

answer $\boxed{x_1(t) = u(t) + r(t-1) - 2r(t-2) + r(t-3) - u(t-4))}$

$-u(t-3)$

1-27 a) $\int e^{3t}\ddot{\delta}(t-2)\,dt = (-1)^2 (3)^2 e^{3t} = 9e^{3t} = \boxed{9e^{6}}$ answer

c) $\int [e^{-3t} + \cos 2\pi t]\dot{\delta}(t)\,dt = (-1)\left[-3e^{-3t} - 2\pi \sin 2\pi t\right]_{t=0}$

$= -1[-3] = \underline{3}$ answer

1-28 a) $10\delta(t) + C_1\dot{\delta}(t) + (2+C_2)\ddot{\delta}(t) = (3+C_3)\delta(t) + 5\dot{\delta}(t) + 6\ddot{\delta}(t)$

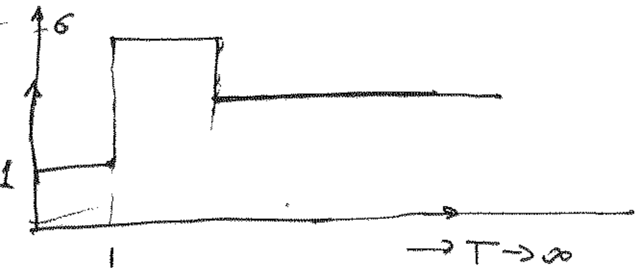
Find $C_1, C_2 \ldots$

Compare the Coefficients

$10 = 3 + C_3 \Rightarrow C_3 = 7$

$C_1 = 5 \Rightarrow C_1 = 5$

$2 + C_2 = 6 \Rightarrow C_2 = 4$



1-38 a) =

$u(t) + 5u(t-1) - 2u(t-2)$

$\frac{1}{2T}\int_{-T}^{T}$

$\lim_{T\to\infty} \frac{1}{T}\left[\int_0^1 dt + \int_1^2 6^2\,dt + \int_2^T 4^2\,dt\right]$

$= \lim_{T\to\infty}\left[\frac{1+36}{T} + \frac{4^2T - 32}{T}\right] = \underline{4^2} = \underline{16} < \infty$ power signal

but as energy signal $\lim_{T\to\infty} 4^2T + 5 \to \infty$ (not energy signal)

c) $\int_0^\infty e^{-10t}u(t) = -\frac{1}{10}e^{-10t}\Big|_0^\infty = \boxed{\frac{1}{10}} < \infty$ energy signal

$(e^{-5t})^2 = e^{-10t} = x^2(t)$ ans

① 1-11 a: $x_a = \cos(10\pi t + \pi/6)$

$\pi/6$ is initial phase does not play role in Computing the Period.

The period is determined from the instantaneous part $10\pi t = 2\pi f_0 t$
$f_0 = 5\ Hz \Rightarrow T_0 = \frac{1}{5}$

b) in the Same way $T_0 = \frac{1}{8.5}$

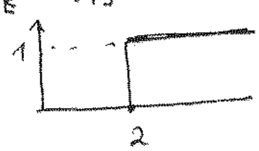
d) $\cos(10\pi t + \pi/6) + 5\cos(17\pi t - \frac{\pi}{4})$
is a Periodic Signal.
Check $\frac{1}{5} / \frac{1}{8.5} = \frac{1}{5} / \frac{2}{17} = \frac{17}{10}$ is integer number

$\frac{17}{10}$. therefore there is a Common period

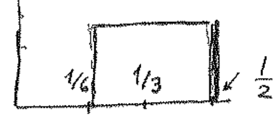
$T_* = \frac{1}{8.5} \times 17 = 10 \times \frac{1}{5} = \underline{\underline{2}}$ is the Common Period.

and the Common frequency is $\underline{\underline{\frac{1}{2}}} = 0.5$

1-16 a) $u[(t-2)/4]$



d) $\Pi(-3t+1)$

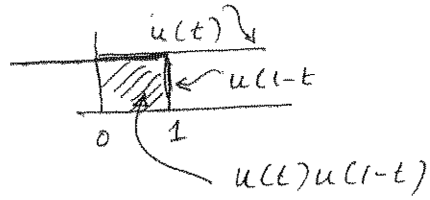


$= \Pi(3(\frac{1}{3} - t)) = \Pi\left[\frac{(\frac{1}{3} - t)}{\frac{1}{3}}\right]$

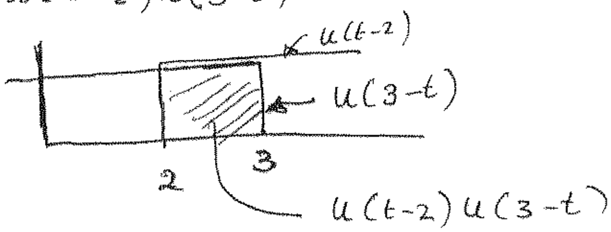
The pulse has a width $\frac{1}{3}$, its Center is at $\frac{1}{3}$ and reversed in time.

1-19 $\sum_{n=0}^{\infty} u(t-2n)\,u(1+2n-t)$

① $n=0 \Rightarrow u(t)\,u(1-t)$

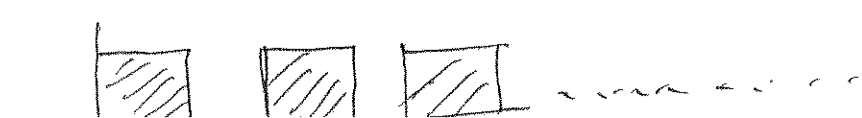


② $n=1$ $u(t-2)\,u(3-t)$

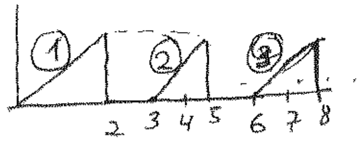


$u(t)u(1-t) + u(t-2)u(3-t)$

=



1-20 a



$$\textcircled{1} = r(t)\,u(2-t)$$
$$\textcircled{2} = r(t-3)\,u(5-t)$$
$$\textcircled{3} = r(t-6)\,u(8-t)$$

$t,\ t-3,\ t-6 \Longrightarrow$ geometric series

$t - 2n$

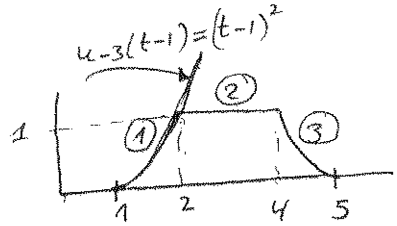
$u(2-t),\ 5-t,\ 8-t$ arithm. series

$\Rightarrow u(2-$

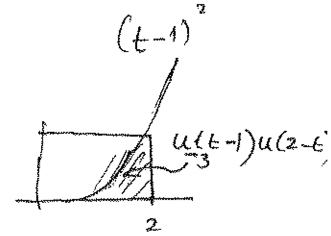
recursive induction:

$$\sum_{n=0}^{\infty} r(t-\cancel{3}n)\,u(2+3n-t)$$

1-21 a

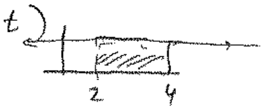


$$(t-1)^2 = u_{-3}(t-1)$$

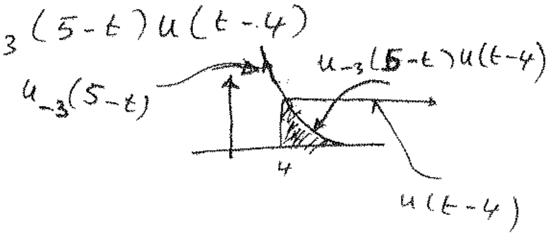


$$\textcircled{1} = (t-1)^2 u(2-t) = u_{-3}(t-1)\,u(2-t)$$

$$\textcircled{2} = u(t-2) - u(t-4) = u(t-2)\,u(4-t)$$



$$\textcircled{3} = (5-t)^2 u(t-4) = u_{-3}(5-t)\,u(t-4)$$



1-23 a

$\delta_\epsilon(t)$ can be realited from

$$\lim_{\epsilon\to 0} \frac{1}{\epsilon} e^{-t/\epsilon}$$

$$\int_{-\infty}^{\infty} \delta_\epsilon\,dt = 1 \overset{?}{=} \int_0^{\infty} \frac{1}{\epsilon} e^{-\frac{t}{\epsilon}}\,dt$$

$$= -\frac{1}{\cancel{\epsilon}}\cdot\cancel{\epsilon}\; e^{-\frac{t}{\epsilon}}\Big|_0^{\infty}$$

$$= \underbrace{-e^{-\frac{\infty}{\epsilon}}}_{0} - \left(-e^{-\frac{0}{\epsilon}}\right)$$

$$= 0 + 1 = 1$$

1-27 a $\displaystyle\int_{-\infty}^{\infty} \underbrace{e^{3t}}_{f(t)} \underbrace{\ddot{\delta}(t-2)}_{\delta^{(2)}(t)} = (-1)^2 f''(2)$

$= (-1)^2 \left(e^{3t}\right)'' \Big|_{t=2}$

$= 9e^{3.1} = 9e^{3.2} = \underline{9e^{6}}$

1-27 c $\displaystyle\int_{-\infty}^{\infty} \Big[\underbrace{e^{-3t} + \cos(2\pi t)}_{f_1(t)+f_2(t)}\Big] \dot{\delta}(t)$

① $\int f_1(t)\,\dot{\delta}(t)\,dt = (-1)^n f_1'(0) = -\left(-3e^{-3.0}\right)$
$= 3 \cdot 1 = 3$

② $\int f_2(t)\,\dot{\delta}(t)\,dt = (-1)^n f_2'(0)$
$= -1\left(-2\pi \sin 2\pi . 0\right) = 0$

$\boxed{① + ② = 3}$

<u>1-28 a</u> $10\,\delta(t) + C_1\,\dot{\delta}(t) + (2+C_2)\,\ddot{\delta}(t)$
$= (3+C_3)\,\delta(t) + 5\,\dot{\delta}(t) + 6\,\ddot{\delta}(t)$

Solution: Compare the Coefficients of the Corresponding base or Coordinate

$10 = 3 + C_3$ from $\delta(t)$

$C_1 = 5$ from $\dot{\delta}(t)$

$6 = 2 + C_2$ from $\ddot{\delta}(t)$

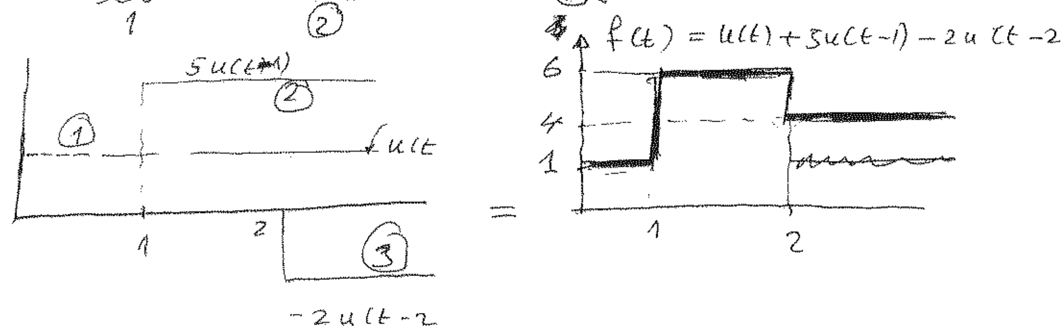
$\Rightarrow C_3 = 7, \quad C_1 = 5, \quad C_2 = 4$

1-38 a

The general definition of Power of a general power signal is:

$$0 < \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} f^2(t)\,dt < \infty$$

$$f(t) = \underbrace{u(t)}_{1} + \underbrace{5u(t-1)}_{2} - \underbrace{2u(t-2)}_{3}$$



$$f^2(t) = 1^2 u(t) + 6^2 u(t-1) + 4^2 u(t-2)$$

$0 \le t \le 1$ $\quad 1 \le t \le 2$ $\quad 2 \le t \le T \to \infty$

$$\lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} f^2(t)\,dt = \lim_{T\to\infty} \frac{1}{2T}\left[\int_0^1 1^2\,dt + \int_1^2 6^2\,dt + \int_2^T 4^2\,dt\right]$$

$$= \lim_{T\to\infty}\left\{\left[\frac{t}{2T}\Big|_0^1\right] + \left[\frac{36t}{2T}\right]\Big|_1^2 + \left[\frac{16t}{2T}\Big|_2^T\right]\right\}$$

$\lim_{T\to\infty}\frac{1}{2T}\to 0$ $\quad \lim_{T\to\infty}\frac{36}{2T}\to 0$

$$= 0 + 0 + \lim_{T\to\infty}\left[\frac{16T}{2T} - \frac{32}{2T}\right]$$

$\to 8$ $\quad \to 0$

$$= 8 \text{ Watt}$$

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c) $$\int_0^\infty e^{-10t}\,dt = -\frac{e^{-10t}}{10}\Big|_0^\infty = \frac{1}{10} \text{ Joule}$$